AFOSR - Unit Combustor

Fundamental Processes that Drive Combustion Instabilities in Liquid Rocket Engines

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1 Introduction

In spite of the research that has been performed over the last half century the reliable design and fundamental knowledge of liquid rocket engines (LRE) is still lacking\cite{1}. High frequency, acoustically coupled combustion instabilities have plagued many LRE design programs leading to costly redesigns or worse, catastrophic failures\cite{2, 3}. In a broad sense, the driving of these acoustically coupled instabilities occurs when energy from the combustion process is added to the acoustic wave in-phase and the net energy addition is greater than damping provided by, for example, viscous dissipation and convection through the nozzle. The specific mechanism for the driving of instabilities can be quite complex and involve phenomena such as acoustics, flow instabilities, interaction of coherent structures, pressure and velocity coupling of combustion, and injection system coupling.

Due to the cost involved in performing full-scale hot fire testing of LRE, the in situ study of combustion stability is prohibitive. Ideally studies could be done on laboratory scale combustors, but many of the mechanisms responsible for instabilities are frequency dependent and hence depend on the physical dimensions of the engine. If a sub-scale combustor was able to exhibit many similar processes at the full-scale frequencies found in real LRE, then more economical research and development could be made. One approach to overcome the difficulty in acoustic scaling is to adjust the acoustic boundary of the sub-scale combustor. By controlling the acoustic impedance of these boundaries the combustor can, at least acoustically, appear similar to the full-scale geometry.

Figure 1 shows two diagram of a typical LRE injector plate from the nozzle end, first, a full-scale LRE and second, a small sector of the same engine with two boundaries actively controlled. While this would be ideal for simulating real LRE, due to the complexity of the acoustics at the boundaries an alternative arrangement was selected. Figure 2 shows a rectangular LRE injector plate, the corresponding sector combustor, and a unit combustor with two acoustically driven walls based on the sector. At low enough frequencies the acoustics at the control boundaries are one dimensional plane waves, both analytically tractable and reproducible with the appropriate acoustic drivers.

![Figure 1: Cylindrical LRE](image)

The current work is to elucidate the driving mechanisms found in unstable LRE. For example how linear or non-linear pressure and velocity coupled combustion response is effected by the intrinsic flow instabilities in the shear layers of the injectors. Specifically the goals of the project
are to develop a sub-scale tool for simulating full-scale acoustically coupled transverse instabilities found in LRE and study the associated driving mechanisms.

2 Theoretical

2.1 Numerical theory and approach: d’ Alembert solution and Heat source model

2.1.1 System model

2.1.1.1 Separate Models for the regions Inside and Outside of the Heat Source

A simplified approach is for theoretical investigation done, instead of approaching the whole phenomena, such as combustion of injected fuel and oxidizer, wave propagations, and interactions between them and other related phenomena, etc. The assumed system can be separated into two regions. The governing equations for both regions are established separately, and combined at the interfaces between them. The heat sources are located in small discret zones, not distributed in the entire domain. The zones are assumed to be acoustically compact. The model of heat source mainly focused on energy relations and not much on acoustics (because it is acoustically compact). Outside the heat source region, the system is assumed such that there is no heat addition, and eventually, an isentropic process could be assumed. This development is performed based on energy relation, linear acoustics theory is assumed to be valid. Thus, the d’ Alembert and 1-D wave solution are valid in these regions.

2.1.1.2 Model of Outside of the Heat Source

Outside of the heat source region, we can use general acoustic solutions (d’ Alembert solution).

- Propagating in $+x$ direction: $f(x - ct)$
Figure 3: Model System

- Propagating in $-x$ direction: $f(x + ct)$

If there is a mean velocity, the solutions are as following.

- Propagating in $+x$ direction: $f(x - (c + U)t)$
- Propagating in $-x$ direction: $f(x + (c - U)t)$

Figure 4: Wave decomposition

2.1.1.3 Model of Inside of the Heat Source

1. Heat Source affects Waves: The wave generation by the heat source is determined by the perturbed energy balance between incoming and outgoing energy by the wave and the heat source fluctuations.
2. Waves and reaction affect heat source: The fluctuation of outgoing and incoming energy through the nozzle and the fluctuation of the energy inside the cavity result in the fluctuation of the net heat addition. This influences on the heat source for generating the waves.
3. Reaction depends on: The heat release by the chemical reaction is influenced by system properties, for example the heat source for the wave generation may be affected by the following factors.

- Chemical Energy accumulation in the Chamber
- Pressure
- Velocity
- The fluctuation of the chemical reaction
2.1.1.4 Governing equations

\[ \dot{y} = (1 - \epsilon)\dot{x}_1 - \epsilon \dot{x}_1 + 2\epsilon \left( \frac{2\gamma}{3\gamma - 1} \right) \dot{Q}'_{RR} \]  

(1)

\[ \dot{Q}'_{RR} = K_H \dot{H}'_\mu + \frac{1}{2} K_P (\dot{y} + \dot{x}_1) + K_u \left( |\dot{u}_0 - \frac{1}{\gamma} \dot{x}_2| - |\dot{u}_0| \right) \]  

(2)

\[ \dot{H}'_\mu = -\dot{Q}'_{RR} \]  

(3)

Where

\[ \epsilon = \left( \frac{1}{2} \frac{\gamma - 1}{c_0} \right) \left[ \frac{\dot{Q}'_{RR,0}}{V} \left( \frac{3\gamma - 1}{2\gamma} \right) \right] \frac{1}{P_0} << 1 \]  

(4)

and

\[ \dot{y} = \dot{g}_1 + \dot{f}_2 \]  

(5)

\[ \dot{x}_1 = \dot{f}_1 + \dot{g}_2 \]  

(6)

\[ \dot{x}_2 = \dot{f}_1 - \dot{g}_2 \]  

(7)

or

\[ \dot{g}_1 = \frac{1}{2} (\dot{y} - \dot{x}_2) \]  

(8)

\[ \dot{f}_2 = \frac{1}{2} (\dot{y} + \dot{x}_2) \]  

(9)
2.2 Numerical results

2.2.1 Numerical Simulation on the Simple Case

The model is simplified to a closed annular tube with 1-D straightened domain. 2-D effects, which are due to the fact the flow turns following the tube, are lost, but the essential physics are kept. The essential physical processes are how the heat sources regions and acoustic waves interact and how instability phenomena formed by the proposed mechanism. For simplicity, we start with single heat source. In the region outside the heat source, d’Alembert solutions work and in the compact heat source region, we use previous equation sets or the combined equation.

![Simple model problem](image)

Figure 6: Simple model problem

2.2.2 d’Alembert Solutions in Space

For the closed circular tube effect, the points, \( i_{\text{min}} \) and \( i_{\text{max}} \), are connected geometrically.

The d’Alembert Solutions are applied in the regions of no heat source: \( i.e. \) in region \( (i_{\text{min}} \sim i_{\text{heat}}) \) and region \( (i_{\text{heat}} \sim i_{\text{max}}) \).

For a given \( \Delta t \) and \( \Delta x \), the above analytical equations can be converted discrete equations suitable for the numerical computation.

\[
\begin{align*}
  f(x - ct) &= f(x - c(t + \Delta t - \Delta t)) = f(x + c\Delta t - c(t + \Delta t)) \\
  g(x + ct) &= g(x + c(t + \Delta t - \Delta t)) = g(x - c\Delta t + c(t + \Delta t))
\end{align*}
\]

If we choose convenient reference value, then the computation becomes more simple.

\[
\Delta t = \frac{\Delta x}{c} \tag{12}
\]

\[
f^n_{i+1} = f^n_{i-1} \tag{13}
\]

and

\[
g^n_{i+1} = g^n_{i+1} \tag{14}
\]

When there is a mean velocity, \( U \), then different characteristic velocities for the \((+)_x\) direction and \((-)_x\) direction should be considered.

Using the above computation, we provide incoming waves \( f_1 \) and \( g_2 \) to the heat source region, and we propagate the calculated waves by the heat source model, \( f_2 \) and \( g_1 \).
2.2.3 1st Order Time Marching for Compact Heat Source Region

As a first simplest form, we use 1st order time marching technique for the heat source model calculations. The specific calculations are omitted, because we keep revise the model and the specific form is also being changed. However, general procedures are not much different. From the incoming waves $f_1$ and $g_2$ to the heat source region, provided by the calculation of outside the heat source region, we can obtained combined term from these information. Lets say $\hat{z}|_{t_n}$.

Using the heat source model, we can calculate the increment of this quantity for the next time step, e.g. $\frac{d\hat{z}}{dt}|_{t_n}$.

The time marching is performed as following.

$$\hat{z}|_{t_{n+1}} = \hat{z}|_{t_n} + \Delta t \frac{d\hat{z}}{dt}|_{t_n}$$

(15)

From $\hat{z}|_{t_{n+1}}$, we can calculate the generated wave, $f_2$ and $g_1$, by the heat source model.

2.2.4 Sample Result 1 (No Mean flow, Traveling to Standing Wave)

The single heat source is located at the center of 1-D straight domain, and the points, $i_{\text{min}}$ and $i_{\text{max}}$, are connected for the closed tube.

There is no mean flow.

The parameters for the simulation are as following.

$$\epsilon = 0.1 << 1, K_H = K_p = K_u = 0$$

(16)

The effect on the heat source model includes only nozzle damping effect.

The initial condition is, at $t = 0$, only sinusoidal traveling wave, propagating in $(+)x$ direction, exists.

Figure 7: Simple model problem 1

In Fig.8, the dashed lines represents the wave fronts. The left figures show the initial behavior of the wave, note it is a traveling wave propagating at the speed of sound. The right figures show the behavior after sufficient time, and the wave fronts are not moving. This means it is a standing wave. When there is no mean flow, a spinning wave, rotating at the speed of sound, gradually transforms into a standing wave.

2.2.5 Sample Result 2 (Mean flow, Standing to Traveling Wave)

The single heat source is located at the center of 1-D straight domain, and the points, $i_{\text{min}}$ and $i_{\text{max}}$, are connected for the closed tube.

There is mean flow.

The parameters for the simulation are as following.
Figure 8: Model 1 results

\[ u_{\text{mean}} = 0.1, \epsilon = 0.1 << 1, K_H = 0, K_p = 0.6, K_u = 1 \]  \hspace{1cm} (17)

The effect on the heat source model includes the nozzle damping effect, the pressure amplifying effect, and the mean flow effect.

The initial condition is, at \( t = 0 \), sinusoidal standing wave exists.

Figure 9: Simple model problem 2

In Fig. 10, the dashed lines represent the wave fronts. The left figures show the initial behavior of the wave, and the wave fronts are moving slowly and oscillating. It is a standing wave being convected at the mean velocity. The right figures show the behavior after sufficient time, and the wave fronts are moving more quickly than the wave fronts of the left figures. In fact, it is a traveling wave propagating at the speed of the sum of the speed of sound and the mean velocity.

When there is mean flow, a standing wave, being convected at the mean velocity, gradually transforms into a spinning wave which propagates at the velocity equal to the sum of the mean
velocity and the speed of sound.

2.3 Wave decomposition analysis for real-time control system

2.3.1 Overview

The goal of the present research is to develop a model that can provide real-time control of the boundary conditions on an impedance tube connected to the combustor of a LRE. Control will be obtained by modifying the current supplied to the acoustic driver. The separation of the plane wave into incident and reflected wave is made by using a two microphone method [4].

2.3.2 Experimental Arrangement

The arrangement (see Fig.11) comprises of an acoustic driver and an impedance tube. Two microphones are installed on the impedance tube at distances $x_1$ and $x_2$. The pressure signals from the microphones are then used to decompose the wave field into incident and reflected waves. The distance between the microphones has to be chosen in such a way to avoid poor calculations.
2.3.3 Analysis

2.3.3.1 Wave Equation Solution

In order to obtain the incident and reflected wave we solve the wave equation for the impedance tube. For this system, a general expression for the acoustic pressure is:

\[ P(x, t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)} \]  

(18)

The boundary conditions are given by the velocity of the membrane of the speaker at \( x = 0 \) and the termination impedance at \( x = L \). In the case of our experiment the velocity of the membrane is given as an input from the speaker model. As for the termination impedance, this is the parameter which will be controlled during the experiment. The boundary condition are given by Eq. 19:

\[
\begin{align*}
  u_0 &= u_+ - u_- \\
  Z &= \frac{P_+ + P_-}{u_+ - u_-}
\end{align*}
\]

(19)

The velocity boundary condition must be transformed into an acoustic pressure condition. This is done via the linearized Euler equation, see Eq. 20:

\[ \rho_0 \frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} \]  

(20)

So:

\[
\begin{align*}
  u_+ &= \frac{A}{\rho_0 c} e^{j(\omega t - kx)} \\
  u_- &= -\frac{B}{\rho_0 c} e^{j(\omega t + kx)}
\end{align*}
\]

(21)

And the input velocity, \( u_0 \), is given by Eq. 22:

\[ u_0 = E e^{j\omega t} \]  

(22)
Substituting Eqs. 21 and 22 into the boundary condition Eq.19, the following expressions are obtained:

\[
\begin{align*}
E &= \frac{A}{\rho_0 c} + \frac{B}{\rho_0 c} \\
Z &= \frac{A e^{-jkL} + A e^{jkL}}{\rho_0 c e^{-jkL} + B e^{jkL}}
\end{align*}
\] (23)

Using Eq.23 the complex amplitudes $A$ and $B$ are obtained for the incident and reflected waves.

\[
A = \frac{(\rho_0 c - Z) E e^{jkL}}{(1 - \frac{Z}{\rho_0 c})2j \sin(kL)}
\] (24)

\[
B = \rho_0 c E - \frac{(\rho_0 - Z) E e^{jkL}}{(1 - \frac{Z}{\rho_0 c})2j \sin(kL)}
\] (25)

### 2.3.4 Computational Method

The electronic separation of a wave field is possible with the arrangement presented in Fig. 12, adapted from [4]. The first microphone, $M_1$, is placed very close to the speaker face while the other, $M_2$, is placed at a specified distance away from the first microphone. The microphones measure the acoustic pressure:

\[
P_1(x, t) = P_i(x, t) + P_r(x, t)
\] (26)

and

\[
P_2(x, t) = P_i(\Delta x, t) + P_r(\Delta x, t) = P_i(x, t + \tau) + P_r(x, t - \tau)
\] (27)

where $\tau = \Delta x/c$ represents the time delay of the signal traveling from the first microphone to the second microphone, and $c$ is the speed of sound. Delaying $P_1(x, t)$ by $\tau$, we obtain

\[
P_{1\tau} = P_1(x, t - \tau) = P_i(x, t - \tau) + P_r(x, t + \tau)
\] (28)

By taking the difference between 26 and 28 we obtain $P_3$

\[
P_3 = P_{1\tau} - P_2 = P_i(x, t - \tau) - P_i(x, t + \tau)
\] (29)

which is proportional the incident wave. Similarly, delaying $P_2$ by $\tau$

\[
P_{2\tau} = P_2(x, t - \tau) = P_i(x, t) + P_r(x, t - 2\tau)
\] (30)

taking the difference between Eqs. 30 and 26, an expression of the reflected wave is obtained.

\[
P_4 = P_{2\tau} - P_1 = P_r(x, t - 2\tau) - P_r(x, t)
\] (31)

\[
P_5 = P_3(x, t - \tau) = P_i(x, t - 2\tau) - P_i(x, t)
\] (32)

The schematic of this method is presented in Fig. 12.
2.3.5 Results from numerical experiment

A Simulink model of the two microphone method was created in order to simulate the impedance measurement on the experimental arrangement described in previous sections. The model takes the acoustic pressure from the two microphones and computes the incident and reflected waves based on the method outlined. The two computed waves are compared with the exact incident and reflected waves calculated from the theoretical solution of the wave equation. Since the original signal from the two microphones has been altered by repetitive subtractions and time delays, we do not expect the exact incident and reflected wave to be equal to the computed one, but rather for the signals to be proportional.

In the numerical experiment the driver was supplied with a chirp signal containing a linearly changing frequency, with a range of 100 to 2000 Hz and various spacings between the microphones were simulated. The results for a spacing of 7 cm is presented in Fig. 13.

As mentioned in the previous section, the two signals are not identical but rather are proportional. A proportionality constant can be found between the two signals. It is seen that the two signals match relatively well in the intermediate frequency range but not as good at low and high frequencies. At distances corresponding to multiples of half wavelength the method does not work well because the information coming from the two microphones is redundant. In order to improve the system an additional microphone may be placed in the tube so there will be appropriate data over a wider frequency range.

2.4 Driver model

2.4.1 Experimental Set-up

A dedicated test rig has been designed and built in order to perform the necessary experiments. It is shown schematically in Fig. 14 and consists of a 2345 mm long acoustic duct of 76 mm in diameter. On one end of the impedance tube there is a magnetodynamic loudspeaker (JBL 2490H). The other end of the duct is a rigid wall. The driving loudspeaker is excited by an amplifier.
Figure 13: Amplitude and Phase absolute errors in frequency spectrum

(LVC 608), driven by a chirp signal produced by a digital function generator. The tested chirp signal’s frequency range is 100 to 1900 Hz. The current requested by the function generator is recorded, as well as the current which the amplifier sends to the speaker.

Figure 14: Schematic of the impedance tube

It can be found the cut-off frequency is 2350 Hz.
Three flush mounted pressure transducers are used to measure the dynamic pressure. To get the best data possible, care must be taken to select the distances between the sensors. The transducers convert the pressure into a voltage which may be amplified and filtered. The tube is also equipped with a thermocouple in order to collect temperature data for the calculation of the speed of sound. All data are are low-pass filtered with a 4th order Butterworth filter with a cut-off frequency of 8 kHz and then, acquired with a sampling rate of 20 kHz by a data acquisition unit (DAQ) composed by a chassis (NI-cDAQ-9172) and a 4-channels analog input module (NI 9215), In the block diagram of Fig.15, the system is shown.

2.4.2 Calibration

Calibration of sensors is necessary in order to adjust for transducers sensitivity (in Volts per psi or Pa). Moreover, the same signal recorded by two different transducers, always shows some mismatches in terms of the resultant voltage, this time-varying discrepancy can be studied in the frequency domain. Referring to Fig.14, consider the two sensors at $x_1$ and $x_2$, assuming $\hat{p}_1$ and $\hat{p}_2$
as the Discrete Fourier Transform (DFT) of the acoustic pressure measured, the ratio:

\[ H_{12} = \frac{\hat{p}_1}{\hat{p}_2} \]  

(33)

indicates the frequency response function (FRF) representing the relative amplitude and phase. Calibration of the transducers system is accomplished by mounting two sensors in a plate at the end of the impedance tube. The sensitivity of one sensor is known approximately (1 psi per 60 mV). In this configuration the second transducer is assumed to be exposed to the same sound filed. If sensor 1 is chosen as the reference, successive comparisons of each additional transducer system with the reference, will result in measurement of the set of FRF \([H_{12c}, H_{13c}, H_{1jc}, ...]\) where the subscript \(c\) refers to the calibration configuration of the sensors. This set of FRF is then used to correct the measured frequency responses according to the following formulas:

\[ \hat{p}_{j\text{corrected}} = [H_{1jc}]\hat{p}_{j\text{measured}} \]  

(34)

Following this procedure for sensors 2 and 3, and ensemble averaging ten data sets, the calibration curves in Fig.16 have been obtained.
The expected calibration functions amplitude has been found to be close to 1.0 and the phase approximately 0.0, as expected.

2.4.3 Characterization of the speaker

The ratio of sampled value of current measured at an integer multiple \(n_i\) of the sampling period \(T_s\), to the corresponding value of velocity through the DFT gives the frequency response function
which links the applied current $I$ to the velocity $U$ of the membrane.

$$TF_{\text{function}} = \frac{DFT(U(n_iT_s))}{DFT(I(n_iT_s))}$$  \hfill (35)

Referring to Fig.15 is possible to see the current is directly recorded in the DAQ, thus, once one gets the vector of data, is possible to obtain the DFT’s using Matlab. To find the pressure, and thus the velocity of the membrane, we must solve the wave equation, which, being a second order differential equation, requires two boundary conditions. Assuming that the transducers are placed on cross sections, $x = x_1$ and $x = x_2$, we define:

$$P_1(t) = p(x_1, t) = [Ae^{-jkx_1} + Be^{+jkx_1}]e^{jwt}$$ \hfill (36)

$$P_2(t) = p(x_2, t) = [Ae^{-jkx_2} + Be^{+jkx_2}]e^{jwt}$$ \hfill (37)

In the frequency domain is possible to write:

$$\hat{p}_1(f) = \hat{p}_1+(f)e^{-jkx_1} + \hat{p}_1-(f)e^{+jkx_1}$$ \hfill (38)

$$\hat{p}_2(f) = \hat{p}_1+(f)e^{-jkx_2} + \hat{p}_1-(f)e^{+jkx_2}$$ \hfill (39)

The Eqs.38 and 39 can be written in an equivalent form consistent with Eqs.36 and 37:

$$\hat{p}_1(f) = \hat{p}(x_1, f) = [\hat{A}e^{-jkx_1} + \hat{B}e^{+jkx_1}]$$ \hfill (40)

$$\hat{p}_2(f) = \hat{p}(x_2, f) = [\hat{A}e^{-jkx_2} + \hat{B}e^{+jkx_2}]$$ \hfill (41)

Defining $s$ as the spacing between the microphones, and $x_1 = 0$, we obtain:

$$\hat{p}_1(f) = \hat{p}(x_1, f) = [\hat{A} + \hat{B}]$$ \hfill (42)

$$\hat{p}_2(f) = \hat{p}(x_2, f) = [\hat{A}e^{-jks} + \hat{B}e^{+jks}]$$ \hfill (43)

The values of $\hat{p}_2$ obtained applying the FFT have to be corrected according to the Eq.34. From to the acoustic theory we define the reflection coefficient as:

$$R(f) = \frac{\hat{p}_2}{\hat{p}_1}$$  \hfill (44)
For a better understanding the system is shown in Fig.17. Using Eqs.33 and 44:

\[
H_{12} = \frac{[\hat{A}e^{-jks} + \hat{B}_1e^{+jks}]}{\hat{A} + \hat{B}} \quad (45)
\]

and because the reflection coefficient in \( x = 0 \) is \( \frac{\hat{B}}{\hat{A}} \), we can write:

\[
R(f)|_{x=0} = \frac{H_{12} - e^{jks}}{e^{jks} - H_{12}} \quad (46)
\]

A tricky aspect of data acquisition has been choosing the right value for \( s \). The appropriate spacing changes in function of the frequency, whose extreme values were \( f_{\text{min}} = 100 \text{ Hz} \) and \( f_{\text{max}} = 2000 \text{ Hz} \). Referring to Eqs.38 and to 39 appears that a condition for the validity of the Eq.46 is that \( \hat{p}_1(f) \) and \( \hat{p}_2(f) \) have to be linearly independent. It can be demonstrated this is equivalent to have \( s \neq n\lambda/2 \) with \( n = 0, 1, 2 \), where \( \lambda \) is the wavelength. Moreover transducers too closely spaced could record almost the same signal because of the presence of noise, and as long as the wavelength is big the minimum spacing for an appreciable difference between the two signals increases.

The compromise between these two limits is reached in an interval \( 0.1 \pi < ks < 0.8 \pi \) that is, \( \frac{0.1\pi}{k} < s < \frac{0.8\pi}{k} \) where \( k = \frac{\omega}{c} \) is the wavenumber. On the other hand, a large spacing will introduce excessive dissipation.

Is it therefore straightforward that the optimal spacing is a function of frequency. Calculations for three values in the range under consideration are reported in Table 1.

<table>
<thead>
<tr>
<th>frequency [Hz]</th>
<th>( \omega ) [rad/sec]</th>
<th>( k ) [rad/m]</th>
<th>( \frac{0.1\pi}{k} ) [m]</th>
<th>( \frac{0.8\pi}{k} ) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>628</td>
<td>1.831</td>
<td>0.0075</td>
<td>0.059</td>
</tr>
<tr>
<td>1000</td>
<td>6280</td>
<td>18.310</td>
<td>0.0171</td>
<td>0.137</td>
</tr>
<tr>
<td>2300</td>
<td>14444</td>
<td>42.110</td>
<td>0.17</td>
<td>1.34</td>
</tr>
</tbody>
</table>

Table 1: Sensor spacing

In our performed measurements, the employment of three transducers made it possible to acquire three data sets from three different couples of transducers with three different spacing between them, the configurations considered have been:

- \textit{configuration}_{12} \rightarrow s_{12} = 6mm
- \textit{configuration}_{13} \rightarrow s_{13} = 12mm
configuration\textsubscript{23} \rightarrow s\textsubscript{23} = 18\text{mm}

The value of the speed of sound has been checked with the succeeding formula: \( c = \sqrt{\frac{\gamma RT}{\rho}} \), using an average value of the recorded temperature.

With an appropriate choice of the reference system, Eq.46 gives the reflection coefficient at \( x = 0 \), thus at a cross section \( x = l \) we have:

\[
R_l = \frac{\hat{B}e^{jkl}}{Ae^{-jkl}} = [R(f)|_{x=0}]e^{2jkl}
\]

(47)

If, for each of the three configurations, we consider \( l \), as the spacing between the rigid wall and the furthest transducer from it:

- \( configuration_{12} \rightarrow l_{12} = 320\text{mm} \)
- \( configuration_{13} \rightarrow l_{13} = 320\text{mm} \)
- \( configuration_{23} \rightarrow l_{23} = 260\text{mm} \)

The \( R_l \) we find is the reflection coefficient of the rigid wall which is a real number with an absolute value of 1, see Fig.(18).

![Figure 18: Rigid wall Reflection Coefficient](image)

From Eqs.44 and 40 we get:

\[
\hat{P}_1(t) = p(x_1, f) = \hat{A}[e^{-jx_1} + Rle^{+jx_1}]
\]

(48)

If now, we assume a new reference system, where \( x = 0 \) is the cross section where the speaker membrane located at equilibrium and, \( x = L \) is the rigid wall, for \( x = x_1 \):

\[
\hat{A} = \frac{\hat{P}_1(f)}{[e^{-jx_1} + Rle^{+jx_1}]}
\]

(49)
To find the acoustic pressure of the membrane we apply the Eq.47 replacing $L$ with $l$. So that:

$$R_L = R_l e^{2jkl}$$  \hspace{1cm} (50)

and:

$$\hat{p}_0(f) = \hat{A}[1 + R_L]$$  \hspace{1cm} (51)

Applying Euler’s Equation in the frequency domain, is possible to express the speed as:

$$\hat{u}_0(f) = \frac{\hat{A}}{Z_0}[1 + R_L]$$  \hspace{1cm} (52)

Where $Z_0 = \rho_0 c$ is the characteristic impedance of the air.

Some results of the application of this procedure are summarized in the figures below:

![Figure 19: The current signal both in the time and in the frequency domain](image1)

(a) The input chirp signal of the current  \hspace{1cm} (b) Amplitude spectra of the current

Figure 19: The current signal both in the time and in the frequency domain

![Figure 20: The acoustic pressure on the membrane](image2)

(a) The pressure signal  \hspace{1cm} (b) Amplitude spectra of the pressure

Figure 20: The acoustic pressure on the membrane

Finally, it has been possible to plot the Bode Diagram, that represents the frequency response function of the real speaker.
2.4.4 The Frequency Response Function of the speaker model

2.4.4.1 The Initial Equation

A simplified model is used to describe the behavior of the membrane for the virtual speaker, here considered as a single DOF system, particularly as a damped harmonic oscillator whose governing equation is:

\[ m \ddot{x} + c \dot{x} + k x = F_{\text{coil}} - p(t)S \]  

(53)

Where:

- \( x, \dot{x}, \ddot{x} \) is the displacement of the membrane from its equilibrium position and its derivatives respect of time;
- \( p(t) \) is the acoustic pressure on the membrane;
- \( S \) is the membrane’s surface area;
- \( m, c, k \) are the mass, damping and stiffness coefficients;
- \( F_{\text{coil}} \) is the force applied by the coil.

The choice to describe the speaker as a one DOF system has been motivated by the fact it is expected the Lorentz driving force will have a frequency close to the first flexural mode of the membrane and far below the second.

From Eq.53, the acceleration can be expressed as:
The acoustic pressure of the membrane and its speed are linked by the input acoustic impedance $Z_0$, where:

$$Z_0 = \frac{p}{u}|_{x=0}$$ (55)

For the system described in Fig.22 the mechanical impedance is:

$$\frac{Z_{m0}}{\rho_0cS} = \frac{Z_{mL}\rho_0cS + jtan(kL)}{1 + j(Z_{mL}\rho_0cS)tan(kL)}$$ (56)

If the pipe is driven at $x = 0$ and closed at $x = L$ by a rigid cap, $Z_{m0}$ is found by the Eq.56 with $Z_{mL} = \infty$ and becomes:

$$\frac{Z_{m0}}{\rho_0cS} = -jcot(kL)$$ (57)

The resonance occurs when $cot(kL) = 0$, that is when $k_nL = (2n - 1)\pi/2$ with $n = 1, 2, 3,...$.

Substituting the Eq.57 in Eq.55 is possible to find the pressure once one knows the speed, so that the Eq.54 becomes:

$$\ddot{x} = \frac{-c\dot{x} - kx + F_{coil} - p(t)S}{m}$$ (58)

Rearranging:

$$\ddot{x} = \frac{(-c - Z_{m0}S)\dot{x} - kx + F_{coil}}{m}$$ (59)

This expression, has been implemented in Simulink as shown in Fig.23

2.4.4.2 Setting the parameters

At a first glance, solving Eq.59 could seem an easy task because, if the mechanical and electrical parameters were known, it could be brought back to the simple mass-spring-damper equation. The point is that, these parameters are unknown and must be identified, moreover a dependence from the exciting frequency is expected.

2.4.4.2.1 The driving force

Figure 24, shows a schematic representation of the moving coil carrying a current $I$ in a uniform magnetic field $B$, created by the permanent magnet. The coil is attached to the membrane which, vibrating, produces pressure waves. As we can see, the current is perpendicular to the lines of force created by the magnetic field, so the Lorentz force will be directed to y-axis and the expression will be:
\[ F(t) = I(t) \int_{coil} BI(t) \times ds \]  

(60)

Also we have to consider the contribute of the backward electromotive force (BEMF); under the influence of this force the electrons move creating a potential difference expressible as:

\[ V(t) = -\int_{coil} u(t) \times B dl \]  

(61)

or alternatively, the voltage can be red in terms of current:

\[ I(t) = -\frac{\int_{coil} u(t) \times B dl}{Z_e(f)} \]  

(62)

Where \( Z_e(f) \) is the coil electrical impedance. This impedance is usually provided by the manufacturer in a form as a graph of impedance versus frequency, so the data must be interpolated from values off a graph. The induced current we obtain from Eq.62 must be multiplied by the B-field according to the Eq.60. Finally, the expression for the \( F_{coil} \) of Eq.59 becomes:

\[ F_{coil} = \left( -\frac{\int_{coil} u(t) \times B dl}{Z_e(f)} \right) B + BI(t) \]  

(63)

Figure 23: The implemented model

Figure 24: A schematic view of the magnetic field in the speaker
2.4.4.2 The stiffness  In an attempt to evaluate the stiffness of the coil $k$, a FEM analysis has been carried out. Assuming linear behavior, a pressure of 10 MPa has been applied in correspondence to the support where the coil applies the pressure to the membrane. With this simulation if the geometry was known, applying a force we would be able to evaluate the displacements and the stiffness of the membrane. With a trial geometry derived by the speaker data sheet, the results of Fig. 30 have been obtained.

<table>
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<tr>
<th>Diameter [m]</th>
<th>Thickness [m]</th>
<th>Material</th>
<th>Density [$Kg/m^3$]</th>
<th>Young modulus [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>0.0008</td>
<td>Titanium</td>
<td>4507</td>
<td>1.1</td>
</tr>
<tr>
<td>m [Kg]</td>
<td>k [N/m]</td>
<td>c [Ns/m]</td>
<td>B [Tesla]</td>
<td>Number of turns</td>
</tr>
<tr>
<td>0.04</td>
<td>8310567</td>
<td>2 $\cdot$ $10^{-12}$</td>
<td>2</td>
<td>150</td>
</tr>
</tbody>
</table>

Table 2: The characteristics of the speaker model

In order to understand better the dynamic response of the membrane a modal analysis has been performed to find the first four normal modes.

2.4.4.3 Results

![Image](a) The chirp signal of the current

![Image](b) Amplitude spectrum of the current

Figure 25: The time trend and the Amplitude spectrum for the virtual current

Comparisons between Fig.28 and Fig.21 show that, even if based on a simplified geometry and assumptions about the force of the coil, the FRF of the model reproduces quite well the trend of the FRF obtained by measurements.

As a future development, new measurements are scheduled using a very short impedance tube in order to study the behavior of the speaker without influences of acoustic resonances. This will enable us to get more information about the speaker parameters.

3 Experimental

Three experimental rigs have been developed for this investigation. The first two are single injector rigs and the last a nine-injector sector of a theoretical larger injector plate. The simplest rig, the
Single Injector No Acoustic Rig (SINAR), has been used for testing the injector itself and to develop some of the diagnostic techniques which will be used throughout the project. The second single injector rig, the Single Injector Acoustic Rig (SIAR), has two configurations which allow for the development and testing of the active control of the acoustic boundary condition in the transverse direction. The third rig, the Multi-Injector Rig (MIR), will be used to demonstrate the active control in a more realistic environment, i.e. with injector to injector interactions.

### 3.1 Injector design

The design was based on the *smart injector* of previous work [5, 6]. Also, the work of Dranovsky was useful in the detailed design [7]. One major difference from previous work is the four supply paths in the injector, see Fig. 31, two for counter swirling oxidizer injection, one for the introduction of liquid fuel through the center swirler, and lastly an annular gaseous fuel injector between the
counter swirling injectors. This allows for the use of liquid and/or gaseous fuels with the same injector.

The two oxidizer paths have significantly different geometric swirl numbers. When the majority of the oxidizer is supplied to path 1 there is a large azimuthal velocity and hence the injection and related heat release remains very near the injector plate. Conversely, if the majority of the oxidizer is supplied to path 2 the swirl is much less and the injection/heat release is located much further from the injector plate. Hence, by controlling the oxidizer mass flow split, the heat release distribution may be modified.

Figure 28: The Bode diagram of the virtual speaker

Figure 29: The mesh used for the FEM simulation
3.2 Single injector no acoustic rig (SINAR)

3.2.1 Design

The SINAR was built to elucidate the conditions under which the injector exhibited unstable combustion. The injector was placed in an approximately 100 mm diameter quartz tube and an adjustable height exhaust nozzle was used to create the combustion chamber, see Fig. 32a. The variable height nozzle has a high blockage ratio to create a low admittance acoustic boundary condition, but the flow is not restricted enough to raise the pressure of the chamber substantially above atmospheric. The length of the combustion chamber may be adjusted from 75 mm to 250 mm. This allows for large changes in the longitudinal acoustics to which the injector may be exposed. This large frequency range allows for the determination of the inherent sensitivity to longitudinal pressure waves.

3.2.2 Hardware

The assembled SINAR test rig is shown in Fig. 32b.

3.2.3 Testing and results

3.2.3.1 Experimental setup

The SINAR rig was run with up to 3.45 MPa air for the oxidizer and 200 kPa natural gas for the fuel (the injector was not configured for liquid fuel). The high-pressure air was metered through a 1.6 mm diameter critical orifice, then split via a valve between the two injector oxidizer supply paths. The oxidizer swirl orifices in the injector are also critical except when one line is completely shut off by the valve, then only one set of orifices are used.

Diagnostic techniques used were dynamic pressure sensing and high-speed Planar Laser Induced Fluorescence (PLIF) of the hydroxyl radical (OH). Dynamic pressure measurements were made at two locations in the injector plate via Kistler 211 b5 piezoelectric transducers. The signal from the sensors were amplified and low-pass filtered with a 4th order Butterworth filter with cutoff frequency of 8 kHz. The analog signals were then recorded with a National Instruments data acquisition module at 20 kHz. The pressure transducers were recessed from the combustion chamber by approximately 85 mm and were actively cooled via high-velocity air.

![Figure 30: The displacement of the membrane under an arbitrary load](image)
The La Vision OH PLIF system, see Fig. 33a, consists of a 42 W Nd:YAG pump laser (532 nm) pumping a frequency doubled amplified dye laser (282 - 285 nm) able to produce approximately 4 W of UV laser power, and an intensified CCD high-speed camera. The system was run at 10 kHz for all runs. The PLIF system allows for the quantification and spatial distribution of OH radical in a thin sheet formed by the UV laser. OH molecules in this thin sheet absorb photons near 283 nm in wavelength, transitioning to a vibronically excited state. The molecule quickly relaxes, through molecular collisions, to a state near the ground vibrational state (still electronically excited) and then through the process of spontaneous emission a photo of approximately 308 nm is emitted and the molecule is returned to its lower electronic level. Light to the high-speed intensified camera is band-pass filtered to allow these emitted photos to reach the lens. The UV laser was tuned daily for the OH excitation transition at approximately 283.94 nm. Figure 33b shows the combustor running and the laser sheet formed by the UV beam (visible where it passes through the quartz). Details of how the OH PLIF data are used is given next.

3.2.3.2 Edge finding

The hydroxyl radical is a relatively short lived chemical species found in the inner layer of reactions zones and in lower concentrations in the products near the flame; thus the steep gradient from little OH to large amounts is an indicator of the reaction zone location [8, 9]. This motivates the use of
edge finding techniques for eventual flame tracking and heat release estimation. Outlined below is a technique which may be used in the identification of spatial location of the steep gradients for reaction zone marking and statistics.

1. Take the 2-D Fast Fourier Transform (FFT) of image

2. Remove data or mask image in spectral domain outside user specified radius from the spectral origin

3. Take the 2-D Inverse FFT to get image for subtraction from the original image

4. Subtract real part of transformed image from original image

5. Perform a 5 X 5 averaging filter to smooth image


7. Create binary image based on Otsu’s Method threshold

8. Define discontinuity in binary image as the edge

9. Sum the edge image in a user defined region of the image to quantify flame length for each frame

Figure 34 is an example of the image processing for analyzing the PLIF data. During data acquisition 10000 frames, one second worth, of data are obtained. This data record is broken into
ensembles for statistical data analysis. For example the record may be broken into 39 data records of 256 frames (9984 frames) and flame length for each of the 256 frames is used as a time varying quantity and these data are transformed into the spectral domain via a FFT. These data will be discussed in the following paragraphs.

The pressure time history is analyzed in the spectral domain, see Fig. 35a. From these data, data from runs with differing nozzle heights, and acoustic calculations it is possible the identify the origin of the peaks in the spectra. Most peaks may be accounted for via acoustic modes of the combustor, but some appear to be hydrodynamic in nature.

In an attempt to identify the coupling mechanism of the injector, the spectra from flame length calculations is contrasted with the pressure spectra. The noise level from the flame length spectra is fairly high, but the large data sets obtained can help offset this through ensemble averaging of many spectra. A total of 39 spectra were available for a chosen record size of 256 images. The plot of these data is shown in Fig. 35b. Here it is hard to identify clear spectral peaks. Figure 35c shows the ensemble averaged data; distinct peaks are clear near 130, 1000, and 2000 Hz. Additional data reduction is required to compare the results from the two techniques.

### 3.3 Single injector acoustic rig (SIAR)

The SIAR combustor is built on the same injector and injector plate as the SINAR combustor. The rig is built to elucidate the transverse stability characteristics of the injector and more importantly demonstrate the active control of acoustic boundary conditions of transverse acoustically-coupled combustion instabilities. The concept allows for both the natural excitation of relatively low-frequency instabilities due to the large transverse length of the combustor in its non-control
configuration and a shorter length configuration with an acoustic driver and control system to emulate the larger test section (Fig. 36b). The large dimension configuration, Fig. 36a, is composed of three main components: combustion chamber, transverse wave sensing section, and acoustic length adjustment section. The sensing section allows for the acoustic waves to be decomposed into right and left traveling waves. The adjustment section is used to tune the acoustics such that the injector responds and instability is obtained. Once an instability is obtained and characterized, e.g., length, frequency, pressure spectra, OH PLIF data, etc, the adjustment section may be exchanged with the driver section. The driving section relies on a JBL 2940H compression driver, which when control is applied, is supplied with the appropriate signal, not to drive the chamber at the frequency of interest, to control the acoustic impedance at the combustor-sensing section.

Figure 34: Example of edge finding routine for PLIF data
interface. The intent is not to drive an instability but to allow a acoustic instability to naturally occur based on the given input parameters to the control system, \textit{e.g.} theoretical test section length.

Figure 35: Example of spectra

Figure 36: SIAR concept
3.3.1 Design

To simplify the acoustic conditions under which the tests will be performed, an approximately uniform cross sectional area for the entire length of the rig was desired. Additionally for ease and accuracy of diagnostic techniques, a rectangular combustion section was selected. The acoustic driver was necessarily a round cross section, so care was taken to maintain cross sectional area and make a gradual transition in shape. Any non-one-dimensional waves due to this transition should be evanescent, decaying quickly for frequencies well below cutoff. The driver throat diameter is 76.2 mm and the combustor and sensing section dimensions were selected to be 76.2 mm in height and 59.8 mm in width. Windows were designed with a step to make the interior of the chamber step-free and a wall-jet air boundary layer cooling/protection system was incorporated. Figure 37 shows various views of the SIAR design and configuration. Similarly to the SINAR combustor, the SIAR has a subcritical exhaust nozzle to provide nearly zero admittance boundary while not raising chamber pressure appreciably. The rig is capable of utilizing adjustable length or driver sections on either end of the combustor.

![Figure 37: SIAR solid models](image-url)
3.3.2 Hardware

Figure 38a shows the assembled SIAR combustor, sensing section, and driving section and the adjustable length section behind the rig. A closer view of the combustor is shown in Fig. 38b.

![Full rig](image1)

(a) Full rig

![Combustion chamber](image2)

(b) Combustion chamber

Figure 38: SIAR Hardware

3.4 Multi-injector rig (MIR)

3.4.1 Design

The concept of the MIR combustor is essentially the same as the SIAR rig, but incorporates multiple injectors to emulate a rocket engine section better. The same drivers have been selected for the MIR combustor, but the cross sectional area must be larger due to the increased number of injectors. Thus, the design must include an area change in the transverse acoustic path. The configuration shown in Fig. 39 has sensing sections and driver sections on both ends of the combustor. Nine injectors are arranged in a 3 by 3 pattern in the injector plate, see Fig. 40a.
3.4.2 Hardware

Parts are currently being manufactured for the MIR combustor. For example the injector test plate, see Fig. 40b, is nearing completion.
4 Conclusion / Future work

- Continued analytical and numerical analysis to improve development and draw more solid conclusions

- Continued development of speaker model, with additional testing to separate and identify acoustic and mechanical unknowns

- Active impedance control system development based on analytical groundwork and speaker model

- Demonstrate active control of acoustic boundary conditions for transverse acoustic waves on both the SIAR and MIR combustors

- Utilize active control to obtain natural acoustically coupled instability and study the mechanisms which allow the natural instability to occur.
References


